Nancy, June 1–5, 2026

May 22, 2026

Contents

Monday, June 1, 2026

Weak model sets of number-theoretic origin and some of their properties

9:30-10:30

Michael Baake

(Bielefeld University, Germany)

Erdős $\mathcal{B}$-free lattice systems and some of their generalisations find a unified frame in the setting of Meyer's weak model sets of maximal density. This allows to determine key properties such as symmetries, spectra and eigenfunctions. The latter are related to exponential sums and Besicovitch almost periodic measures. We review some of these properties via paradigmatic examples such as visible lattice points or carefree numbers.

(This is partly based on joint work with Uwe Grimm, Christian Huck, Robert Moody, Andreas Nickel and Peter Pleasants.)

U-adic integers for some Pisot U-numerations

11:00-11:25

Olivier Carton[1] and Reem Yassawi[2]

[1]IRIF, [2]Queen Mary University of London

In this work, we introduce, for each Pisot numeration U satisfying a mild condition, a group $\mathbb{Z}_U$ which is the analog of the additive group $\mathbb{Z}_p$ of the p-adic integers. It can be viewed as a topological completion of the set of normalized U-expansions of natural integers, as each element of $\mathbb{Z}_U$ can be represented by an infinite normalized sequence of digits. Our construction of $\mathbb{Z}_U$ does not follow the same strategy as for $\mathbb{Z}_p$ for the following reason. When two base-p expansions are added, the carries propagate to the left. This is reflected in the fact that the p-adic valuation ν_p satisfies the classical inequality $\nu_p(x+y) \geqslant \min(\nu_p(x), \nu_p(y))$. This property no longer hold for Pisot numerations; carries may propagate both to the left and to the right, as is the case for the Zeckendorf numeration. For this reason, our construction is based on a pseudo-valuation ν_U which satisfies the weaker inequality $\nu_U(x+y) \geqslant \min(\nu_U(x), \nu_U(y)) - K$ for some constant K; we say that numerations that satisfy this *preserve zeros*. We also equip our group $\mathbb{Z}_U$ with a topology. As a result of the nonstandard propagation of the carries, $\mathbb{Z}_U$ is not a profinite group with a zero dimensional topology.

Our main result is that for a given Pisot numeration, the group that we construct is also topologically a torus, in the presence of mild hypotheses. In particular, we show

Theorem 1 *Let $U = (u_n)_{n \geqslant 0}$ be a Pisot numeration with standard initial conditions that preserves zeros. Then there are k, ℓ and a continuous group isomorphism $\Phi : \mathbb{Z}_U \to (\mathbb{C}^k \times \mathbb{R}^\ell)/\mathbb{L}$ where $\mathbb{L}$ is a lattice.*

Much of the novelty in our result is the development of the group $\mathbb{Z}_U$, which hinges on the fact that finding the canonical expansion of a sum is computable by a finite automaton, as developed by Frougny.

11:30-11:45

The arithmetic of multidimensional continued fractions

Giuliano Romeo
(Politecnico di Torino)

Continued fractions provide a powerful representation of real numbers from several points of view. An intriguing problem is to also perform efficient computations with continued fraction expansions. For classical continued fractions, Gosper defined an algorithm that successfully achieves this. In this talk, we present algorithms to perform efficient computations with multidimensional continued fractions. Multidimensional continued fractions were introduced by Jacobi in order to generalize the results from the classical theory of continued fractions. We focus on the computation of the Möbius and the bilinear fractional transformation of multidimensional continued fractions arising from matrix algorithms, such as Jacobi-Perron, Brun, Selmer, Euler, and Poincaré. The talk is based on joint work with P. Miska and N. Murru.

11:50-12:05

The arithmetic of continued fractions in the field of p-adic numbers

Giulia Salvatori
(Politecnico di Torino)

Continued fractions have been long studied due to their strong properties, such as rational approximation. In this extent, their arithmetic over real numbers has represented an intriguing problem throughout the years. In this talk, we present the arithmetic of continued fractions over the field of p-adic numbers. In particular, we provide a complete methodology to compute the p-adic continued fraction of the Möbius transformation and the bilinear fractional transformation of p-adic numbers. These allow any standard arithmetic operation over p-adic numbers to be performed. In great contrast with real continued fractions, we prove that the knowledge of arbitrarily many partial quotients of the initial continued fractions is not always sufficient to recover some partial quotients of the transformations. However, we show that the set of elements for which this is not possible has Haar measure zero in $\mathbb{Q}_p$.

Joint work with Giuliano Romeo.

Pentagonal and decagonal quasiperiodic point sets

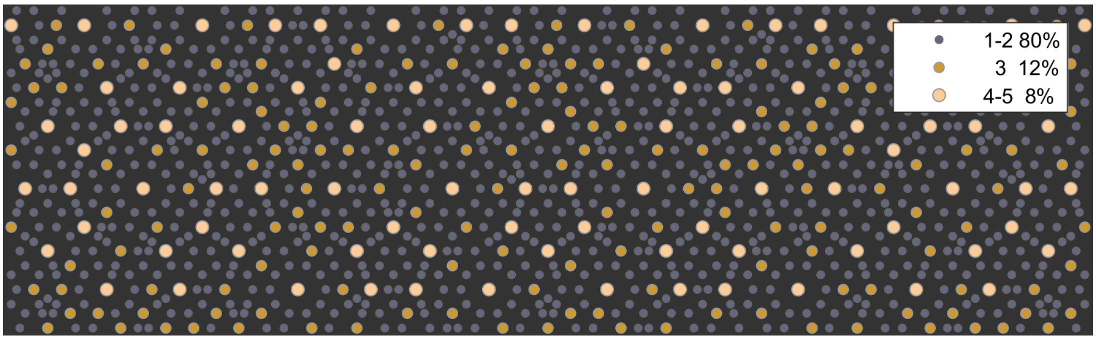
12:10-12:25

Christoph Bandt[1] and Yves Meyer[2]

([1]Universität Greifswald, [2]ENS Paris-Saclay)

Let R denote the ring of algebraic integers either of a cyclotomic field containing a real Pisot unit β, or of the field generated by a complex Pisot unit β. Consider an iterated function system (IFS) $g_k(z) = \beta z + z_k,\ k = 1, ..., m$ with z_k in R. We assume that the attractor A of a conjugate IFS $f_k(z) = \beta' z + z'_k$ has interior points. This can be forced by taking sufficiently many mappings. Then there is a unique cut-and-project model set Λ which fulfils $\Lambda = \bigcup g_k(\Lambda)$.

We present an algorithm which directly determines the set Λ, avoiding the difficulties with the fractal structure of the attractor A. Classical constructions are based on tiles A of different shape. Hejda and Pelantova (2016), and Hare, Masakova and Vavra (2018) worked with overlapping attractors. In our approach, the overlaps provide a natural decoration of Λ. The method is illustrated for the pentagonal case where $\beta \approx 1.618$ and the z_k are the fifth roots of unity.



Completeness of polygonal numeration systems

14:15-14:40

Zuzana Masáková and Milena Svobodová

(Czech Technical University in Prague, FNSPE)

We study polygonal numeration systems, where the base is a real number $\beta > 1$ and the alphabet of digits is formed by the n-th roots of 1 and the origin. Using the notion of the spectrum of numbers, as defined by Erdős, we provide a criterion based on the size of the base β deciding whether every complex number has a representation in such a system. We comment on the properties of the spectrum and arithmetical aspects of such systems.

14:45-15:00

Transcendence criteria for p-adic and multidimensional continued fractions

Nadir Murru

(Politecnico di Torino)

Classical results in Diophantine approximation, such as Roth's theorem, provide powerful tools for proving the transcendence of real numbers defined by special classes of continued fractions. These techniques can be adapted to other settings, notably to the p-adic context and to generalizations such as multidimensional continued fractions. However, in both cases some obstructions arise, preventing a straightforward extension of all results known in the classical real case. In this talk, we survey the current state of the art and present recent advances concerning p-adic and multidimensional continued fractions. In particular, we prove that palindromic and quasi-periodic p-adic continued fractions converge either to quadratic irrationals or to transcendental numbers, removing any restriction on the p-adic norm of partial quotients that were required in previous works. We also establish transcendence results for certain Liouville-type and quasi-periodic multidimensional continued fractions.

15:05-15:20

Superoptimal continued fractions

Slade Sanderson

(IRIF)

For a.e. irrational x, any semi-regular continued fraction (SRCF) expansion of x with convergents P_k/Q_k satisfies (i) $\sup_k Q_k|Q_k x - P_k| \geq 1/2$ and (ii) $\limsup_k n(k)/k \leq \log 2/\log\varphi$, where $\varphi = (\sqrt{5}+1)/2$ and $n(k)$ is such that $q_{n(k)} \leq Q_k < q_{n(k)+1}$ with p_n/q_n the n^{th} regular continued fraction convergent of x. In 1987, Bosma introduced an algorithm producing *optimal continued fractions*, which provide both the 'best approximations' and the 'fastest convergence' among SRCFs: these satisfy (i) $Q_k|Q_k x - P_k| < 1/2$ for all k and (ii) $\lim_k n(k)/k = \log 2/\log\varphi$ a.s.

In this talk, we show that—within a broader class of generalised continued fractions (GCFs) which contain SRCFs—the OCFs can be improved. In particular, for any choice of $\varepsilon, C > 0$, we construct a whole family of dynamical systems which, for a.e. x, produce *superoptimal* GCF-expansions whose convergents satisfy (i) $Q_k|Q_k x - P_k| \leq \varepsilon$ for all k and (ii) $\lim_k n(k)/k = C$. Among these, we consider two interesting, concrete examples—the *Fibonacci* and *Hurwitz–Borel* continued fraction algorithms—which produce a GCF of *each* irrational x and which improve on the bounds of the OCFs.

Random Coverings and Littlewood's Conjecture

15:50-16:05

Agamemnon Zafeiropoulos
(Technical University Graz)

Let $\mathcal{K} = \{\alpha \in [0,1] : \sup_{n \geqslant 1} \frac{1}{n} \log q_n(x) < \infty\}$. We show that given $\alpha \in \mathcal{K}$ and $\gamma \in \mathbb{R}$, there exists a set $\mathbb{G} = \mathbb{G}(\alpha, \gamma) \subseteq [0,1]$ of Hausdorff dimension $\dim_{\mathrm{H}} \mathbb{G} = 1$ such that for any $\beta \in \mathbb{G}$, the following holds: For any $\delta \in \mathbb{R}$, we have

$$q\|q\alpha - \gamma\|\|q\beta - \delta\| < \frac{1}{\log q} \text{ for inf. many } q \geqslant 1.$$

This is joint work with Hauke, Schubin and Stefanescu.

On Conway's Numbers and Games

16:10-16:25

Wolfgang Bertram
(Université de Lorraine)

In this small talk, I would like to invite you to a short journey, as far as possible in 15 minutes, into the realm of "surreal" numbers. John Horton Conway discovered the later so-called surreal numbers almost by accident, when working on combinatorial games ([Co01], cf. [He91]). Until today, these numbers are hardly known in the mathematical community, for a couple of reasons. One of the goals of my preprint [Be25] was to change this situation and to propose an approach (influenced by Gonshor's [Go86]) which is sufficiently close to "mainstream mathematics". This approach also sheds new light on the link between "quantum" and "classical" mathematics – whereas the usual theory of real numbers may be considered as "classical", Conway's numbers clearly have a "quantum flavor". I think there should also be links between Conway's reals and the theory of *q-rationals and q-reals* recently developed by Ovsienko and Morier-Genoud (see [MGO22]).

From a more philosophical or foundational point of view, Conway's numbers are an important modern contribution to answer Dedekind's question (1888), *What are and what should be the numbers?* This kind of answer refers both to Set theory and to Type theory, and we propose a unification grounded on Conway's numbers, to be called "Numbered Set Theory".

Be25 Bertram, W., "On Conway's Numbers and Games, the Von Neumann Universe, and Pure Set Theory". `https://arxiv.org/abs/2501.04412` (extended and revised version to appear)

Co01 Conway, J.H., *On Numbers and Games* (Second Edition), A K Peters Ltd., Wellesley 2001

Go86 Gonshor, H., *An Introduction to the Theory of Surreal Numbers*, London Math. Soc. LNS **11**, Cambridge University Press, Cambridge 1986

He91 Hermes, H., "Numbers and Games", Chapter 13 in: Ebbinghaus et al., *Numbers*, Springer (translated from *Zahlen*), `http://www.maths.ed.ac.uk/%7Eaar/papers/numbers`

MGO22 Morier-Genoud, S., and V. Ovsienko, "On *q*-deformed real numbers", Experimental Mathematics Volume 31, 2022 - Issue 2, `https://arxiv.org/abs/1908.04365`

Tuesday, June 2, 2026

Weighted products of digits for Gauss-like maps

9:30-10:30

Ayreena Bakhtawar

(Polish Academy of Sciences, Poland)

Given integers $i_1 < \cdots < i_k$, a parameter $B > 1$, and exponents $t_1, \cdots, t_k > 0$, we study the set of real numbers x whose continued fraction digits $(a_n(x))_{n\geq 1}$ satisfy

$$\prod_{j=1}^{k} a_{n+i_j}(x)^{t_j} \leq B^n$$

for infinitely many n. We determine the Hausdorff dimension of this set, extending and refining earlier results in the literature, whose motivation goes back to work of Kleinbock and Wadleigh.

We further generalize these results to the setting of d-decaying Gauss-like maps, introduced by Liao and Rams, where the digits $(a_n(x))_{n\geq 1}$ arise from the natural symbolic coding of the system. To treat this class, we develop elements of thermodynamic formalism beyond the standard framework for infinite iterated function systems due to Mauldin and Urbański, as their acceptability condition is not satisfied in our setting.

This is joint work with Michał Rams .

Algebraic properties of substitution words over infinite alphabets

11:00-11:25

Dirk Frettlöh

(Bielefeld University)

Today a lot is known about aperiodic tilings of the line, or about aperiodic infinite words. Usually they use a finite number of distinct building blocks (tiles, resp., letters). Only recently Manibo, Rust, and Walton developed a theory that allows a meaningful study of aperiodic words over infinite alphabets with respect to their dynamical and ergodic properties. This talk explores some of the new phenomena that occur in the case of substitutions over infinite alphabets.

11:30-11:45

Another Naturally Appearing Family of Cantorvals

Petra Staynova

(University of Derby)

A *Cantorval* is an intriguing topological subset of $\mathbb{R}$ that combines the properties of both an interval and a Cantor set. While Cantorvals have been observed in the study of sumsets of Cantor sets, their systematic construction through digit systems remains relatively unexplored.

In this paper, we investigate a family of self-similar sets generated by *standard primitive digit systems* $(Q, \mathcal{D})$. We focus specifically on the family $Q = 3$ with digit sets $\mathcal{D} = \{-1, 0, 3k + 1\}$ for $k \in \mathbb{N}$. We classify the topology of these digit tiles, and prove that if the digit set $\mathcal{D}$ consists of consecutive integers, the resulting tile is an interval; otherwise, it is a symmetric Cantorval.

Using the contact matrix method of Grochenig and Haas, we provide a closed-form representation for the matrices describing the boundary of these tiles. We calculate the Hausdorff dimension of the boundary, $\dim_H \partial t_k$, and provide numerical evidence about the behavior of the dimension as k approaches infinity. This work provides a new, constructive framework for generating and analyzing Cantorvals in the context of fractal tilings.

This is joint work with Natalie Frank, Jan Mazac, May Mei, and Kitty Yang.

11:50-12:05

From cocompact Fuchsian groups to chaotic Delone sets

Tony Samuel

(University of Birmingham, UK)

Motivated by the question, *can developing new cut and project models, where the lattice is not square or the curve is non-linear, generate better performing graded metamaterials?* raised by Davies *et al.* [*Phys. Rev. Lett.* **131**, 2023], in this talk we will discuss the construction of cut and project schemes in the Poincaré disk model of hyperbolic space. We will present conditions on the construction so that the resulting point set is (chaotic) Delone, and moreover, discuss results pertaining to the set of distances (tile lengths). We will conclude with some applications of these results to cocompact Fuchsian triangle group, complementing the work of Álvarez López *et al.* [*Discrete Contin. Dyn. Syst.* **41** (2021)]. This is joint work with Richard Howat (University of Birmingham, UK) and Ayşe Yıltekin-Karataş (Bartın University, Turkey).

The Minimal Denominator Problem in Function Fields 12:10-12:25

Noy Soffer Aranov

(Technical University Graz)

Meiss and Sanders proposed an experiment in which they fix $\delta > 0$ and study the statistics of the minimal denominator Q for which there exists a rational $\frac{P}{Q} \in (x-\delta, x+\delta)$, where x is varied. Discrete analogues of this question where also studied by Kruyswijk and Meijer, Balazard and Martin, and Shparlinski. In this talk, I will discuss the history of this problem and its generalizations, as well as the function field analogue of this question. We compute the continuous distribution in the function field setting using linear algebra. Moreover, we use ultrametricity to prove that unlike the real setting, the discrete and continuous are identical, even when the denominators are restricted to subsets of the ring of polynomials. This is based off the papers `https://arxiv.org/pdf/2501.00171` and `https://arxiv.org/abs/2510.07787`.

On Positivity of Nearly Linearly Recurrent Sequences 14:15-14:40

James Worrell

(University of Oxford)

Nearly linear recurrences are a generalisation of linear recurrences and are instances of linear time-invariant systems in control theory and linear constraint loops in program analysis. In this talk we formulate the Positivity Problem for such recurrences. This asks whether all sequences satisfying a given recurrence with given initial conditions are positive. This problem is a generalisation of the Positivity Problem for linear recurrence sequences, and is a special case of the non-reachability problem for linear time-invariant systems. We describe is a decision procedure for the Positivity Problem for recurrences of order two. The termination proof of our procedure relies on a new transcendence result for infinite series that is of independent interest.

This talk is based on joint work with Amaury Pouly and Mahsa Shirmohammadi.

14:45-15:00

The lazy algorithm for reflective numeration systems

Benoît Rittaud
(Université Paris-13)

Consider a sequence $\mathcal{R} := (x \mapsto 2u_i - x)_{i\in\mathbb{N}}$ of reflections. With some natural assumptions on $(u_i)_{i\in\mathbb{N}}$, any integer $n \in \mathbb{N}$ can be represented by a binary word g_n that codes a succession of elements in $\mathcal{R}$ such that, when these reflections are successively applied starting from n, a decreasing sequence of integers is obtained whose last term is equal to 0. The extremal case of such a *reflective numeration system* is the one for which $u_i = 2^i - 1/2$: in this case, g_n is unique, and $(g_n)_{n\in\mathbb{N}}$ is exactly the standard binary Gray code. This provide a "local" definition of it, alternative to the usual "global" one given by increasing sequence of lists. From this "local" definition is easily proved the seminal *flipping digit property*: consecutive elements of $(g_n)_{n\in\mathbb{N}}$ differ by exactly one digit.

For other choices of $\mathcal{R}$, the words coding n are not unique in general. For the lazy algorithm, that maps n to the radix-biggest binary word l_n coding it, we have that the sequence $(l_n)_{n\in\mathbb{N}}$ always satisfies the flipping digit property. Choosing $\mathcal{R}$ conveniently provides sequences of binary words that satisfy both the flipping digit property and some combinatorial constraints. The cases of Fibonacci and k-bonacci constraints were done in a "global" way in [A. Bernini, S. Bilotta, R. Pinzani, and V. Vajnovszki, *Two Gray Codes for q-ary k-generalized Fibonacci Strings*, Proceedings of ICTCS 2013, 54–59].

Links are also to be made with some binary trees, noticeably trees with Fibonacci combinatorics and a tree whose structure embedds naturally the one of *knock-out tournaments* and the Narayana-Zidek-Capell sequence.

15:05-15:20

New properties of the φ-representation of integers

Ingrid Vukusic
University of Waterloo

We prove a few new properties of the φ-representation of integers, where $\varphi = (1+\sqrt{5})/2$. In particular, we prove a 2012 conjecture of Kimberling. As software assistants, we used the `Walnut` theorem-prover, and in one proof, `ChatGPT 5`.

Joint work with Jeffrey Shallit. https://arxiv.org/abs/2509.16150

State Complexity of Shifts of the Fibonacci Word 15:50-16:05

Pierre Popoli
(University of Waterloo)

The Fibonacci infinite word $\mathbf{f} = (f_i)_{i \geq 0} = 01001010 \cdots$ is one of the most celebrated objects in combinatorics on words. There is a simple 5-state automaton that, given i in lsd-first Zeckendorf representation, computes its i'th term f_i, and a 2-state automaton for msd-first. In this paper we consider the state complexity of the automaton generating the shifted sequence $(f_{i+c})_{i \geq 0}$, and show that it is $O(\log c)$ for both msd-first and lsd-first input. This is close to the information-theoretic minimum for an aperiodic sequence. Moreover, we also prove a close formula for the state complexity in the lsd-first case, which implies that the function that maps c to the state complexity of $(f_{i+c})_{i \geq 0}$ is Fibonacci-regular. The techniques involve a mixture of state complexity techniques and Diophantine approximation.

This is a joint work with D. Moradi, J. Shallit and I. Vukusic.

Integer Linear Optimization in the Search for Integer Polynomials with Prescribed Properties 16:10-16:25

Jean-Marc Sac-Épée
(Université de Lorraine)

Many problems in algebraic number theory amount to searching for integer polynomials satisfying prescribed properties. Although the questions arising in this context are of very different nature, they all involve very large search spaces that cannot be explored by direct methods.

In this talk, I will present an approach based on integer linear optimization that makes it possible to treat these problems within a unified framework. The idea is to translate the prescribed properties into linear constraints on the coefficients, thereby reformulating the original problem as a discrete optimization problem. This approach makes it possible to organize the exploration of large families of polynomials and to produce explicit new examples having the prescribed properties.

I will illustrate this method through several applications: the search for polynomials with small span, the construction of Salem numbers of trace -3 and minimal degree, the search for small Salem numbers, the study of polynomials of small length, the study of Newman polynomials and their divisors, and certain variants of the Prouhet–Tarry–Escott problem.

Beyond the specific results obtained in each of these contexts, the aim of this talk is to show that integer linear optimization provides a flexible, efficient, and unifying tool for the study of a wide variety of problems.

Wednesday, June 3, 2026

Expansions in Multiplicatively Independent Bases : an Automata-Theoretic Point of View

9:30-10:30

Colin Faverjon

(University of Picardy Jules Verne, Amiens, France)

The decimal expansion is the most commonly used base in Western society for representing numbers, while the binary expansion is ubiquitous in technological environments. Although billions of conversions between these two representation systems are performed daily, deep mysteries persist regarding these conversions. Consider, for example, the following two statements, both of which remain unresolved:

(a) Your PIN code appears in the decimal expansion of every sufficiently large power of 2;

(b) Your birth date appears infinitely many times in the decimal expansion of the real number whose binary expansion

$$.010\,100\,010\,100\,010\,010\,010\,100\,010\,\ldots$$

is the unique fixed point of the morphism $0 \mapsto 010$, $1 \mapsto 100$.

Both statements rely on the heuristic that expansions in multiplicatively independent bases (such as 2 and 10) should exhibit no common structure. Furstenberg formalized this heuristic through a series of results and conjectures concerning the joint behavior of the dynamical systems $\times p$ and $\times q$ on $\mathbb{R}/\mathbb{Z}$.

In this talk, we approach this question from the perspective introduced by Turing, Hartmanis, Stearns, and Cobham: that of computational complexity. One of the simplest classes of Turing machines is that of deterministic finite automata. Since powers of 2 are recognizable by a finite automaton from their binary expansion, Cobham's theorem implies that no automaton can recognize them from their decimal expansion, providing a partial answer to (a). A longstanding conjecture, related to (b), posited that a similar conclusion holds for real numbers: only rational numbers possess the property that their expansions in two multiplicatively independent bases can be produced by finite automata.

This conjecture has been recently resolved using a transcendence method known as Mahler's method. This approach also provides a new proof and a large generalization of Cobham's theorem.

11:00-11:25

Intersection of Sparse Automatic Sets in Multiplicatively Independent Bases

Seda Albayrak
(Simon Fraser University)

In 1844, Catalan conjectured that the Diophantine equation $2^n + 1 = 3^m$ has only finitely many solutions. Motivated by such questions, we study intersections of sets arising from distinct multiplicative structures. We prove that if k and ℓ are multiplicatively independent integers, then any sparse k-automatic set has finite intersection with any sparse ℓ-automatic set. From the automata-theoretic perspective, sets such as $\{2^n + 1 : n \geq 0\}$ and the powers of 3 are sparse automatic sets in different bases. Our result applies far beyond these simple examples: sparse k-automatic sets can exhibit significantly more complicated structure than pure exponential forms. We also establish a multidimensional version for subsets of $\mathbb{N}^d$, and provide effectively computable upper bounds on the size of the intersection in terms of the data of the automata accepting these sets. Finally, we explain how these results relate to a conjecture of Erdős. This talk is based on joint work with Jason Bell.

11:30-11:45

Parry condition versus existence and uniqueness of alternate bases

É. Charlier[1], S. Kreczman[1], Z. Masáková[2], E. Pelantová[2] and P. Šťovíček[2]
([1]Université de Liège, [2]Czech Technical University, Prague)

A p-alternate numeration system is given by a base $\mathcal{B} = (\beta_n)_{n\in\mathbb{N}}$ of numbers $\beta_n > 1$ which satisfy $\beta_n = \beta_{n+p}$ for every $n \in \mathbb{N}$. The $\mathcal{B}$-expansion of $x \in [0, 1)$ is the lexicographical greatest sequence $(a_n)_{n\in\mathbb{N}}$ of non-negative integers such that $x = \sum_{n=0}^{+\infty} \frac{a_n}{\beta_0\beta_1\cdots\beta_n}$. To such a base $\mathcal{B}$ there is assigned a list of the quasi-greedy expansions of unity $\mathbf{d}^{(\ell)} = d_0^{(\ell)} d_1^{(\ell)} d_2^{(\ell)} \cdots$, with $\ell = 0, 1, \ldots, p-1$. The list allows to decide whether $(a_n)_{n\in\mathbb{N}}$ is a $\mathcal{B}$-expansion of some $x \in [0, 1)$. For each $\ell = 0, 1, \ldots, p-1$, the list satisfies the so-called

$$\text{Parry condition} \quad \begin{cases} d_n^{(\ell)} \in \mathbb{N} \ \text{ for every } \ n \in \mathbb{N}, \\ d_0^{(\ell)} > 0 \ \text{ and } \ d_n^{(\ell)} > 0 \ \text{ for infinitely many } \ n \in \mathbb{N}, \\ d_n^{(\ell)} d_{n+1}^{(\ell)} d_{n+2}^{(\ell)} \cdots \preceq_{lex} \mathbf{d}^{(\ell+n \mod p)} \ \text{ for every } \ n \in \mathbb{N}. \end{cases}$$

We address the inverse question, whether a list that satisfies the Parry condition corresponds to the list of quasi-greedy expansions of unity for some base $\mathcal{B}$ and whether such a base is unique. We answer this question in the affirmative. If $p = 1$, the reasoning is easy as the numeration system coincides with the system introduced by Rényi. In the case of larger p, our proof relies on Furstenberg's theorem and a result of Gale and Nikaido from 1965.

Measures arising from regular sequences

11:15-12:05

Neil Manibo

(Universität Bielefeld)

In this short talk, we will describe how to construct (a parametrised family of) measures from a regular sequence. The linear recursions satisfied by the sequence allow one to prove explicit properties of these measures. In particular, we will describe how spectral properties of the measures are related to asymptotic growth behaviour of the underlying regular sequence. This is based on joint work with Michael Coons, James Evans, and Philipp Gohlke.

Asymptotics of Regular Sequences

12:10-12:25

Clemens Heuberger

(University of Klagenfurt)

The asymptotics of the summatory function of regular sequences—a class of sequences which is basically defined by matrix products depending on a digit expansion—has been studied extensively. In many cases, the sequence itself fluctuates too much in order to admit a direct analysis and it is only the summatory function where an asymptotic analysis makes sense. In some cases, however, the sequence itself admits a precise asymptotic expansion, for example in the case of sequences related to divide and conquer algorithms. In this talk, sufficient conditions (based on the eigenspaces of the matrices involved) for regular sequences admitting a precise asymptotic analysis are presented.

Thursday, June 4, 2026

Automatic sequences in rational bases and the particular case of Thue-Morse in base 3/2

9:30-10:30

Manon Stipulanti

(University of Liège, Belgium)

In combinatorics on words (and other related fields), the Thue-Morse sequence is ubiquitous. It is a classical example within the family of so-called "automatic sequences". These sequences are produced by finite automata reading representations of integers in some numeration systems. In its most general form, a numeration system is a way of representing integers thanks to a language. These were called "abstract numeration systems" by Lecomte and Rigo in 2001. When the underlying language is regular, the corresponding automatic sequences have been well understood. But what happens when the language is not regular?

In this talk, we consider rational bases for which the numeration language is (highly) not regular and which were introduced by Akiyama, Frougny, and Sakarovitch in 2008. Nevertheless, we study automatic sequences in this context and obtain general results on them such as a Cobham-like theorem. We also show that these sequences can be obtained by alternating several morphisms (or substitutions) instead of applying only one as in the regular case. Such a construction resembles that leading to the Oldenburger-Kolakoski word, for which the question of determining its letter frequencies is still open. By considering the Thue-Morse sequence in base 3/2 (whose nth term is given by the sum modulo 2 of the digits in the base-3/2 representation of n), we look at and answer similar combinatorial questions: Is this word (uniformly) recurrent? What is its factor complexity? How symmetric is its set of factors? What are its letter frequencies?

This talk is based on a blend of works with Michel Rigo on the one hand, Julien Cassaigne, Bastián Espinoza, and Michel Rigo on the other.

11:00-11:25

Construction of normal numbers for dynamically generated Cantor series and an effective hotspot theorem

Dylan Airey[1], Bill Mance[2], Diego A. Rojas[3]

([1]Princeton University, [2]Uniwersytet im. Adama Mickiewicza w Poznaniu, Poznań, [3]Sam Houston State University)

Let any $Q = (q_n) \in \mathbb{N}_{\geq 2}^{\mathbb{N}}$ be called a *basic sequence*. The *Q-Cantor series expansion* of a real number x is the (unique) expansion of the form

$$x = a_0 + \sum_{n=1}^{\infty} \frac{a_n}{q_1 q_2 \cdots q_n} \tag{1}$$

where $a_0 = \lfloor x \rfloor$ and a_n is in $\{0, 1, \ldots, q_n - 1\}$ for $n \geq 1$ with $a_n \neq q_n - 1$ infinitely often.

Farhangi and Mance previously studied normality in the context of dynamically generated basic sequences. This class includes basic sequences Q such as translated Thue-Morse sequences and translated Chamerpnowne sequences. Under some minor additional conditions, it's known that Q-normality and Q-distribution normality are equivalent if Q has zero entropy.

For a given Q in this class of basic sequences, we provide a construction of a number that is both Q-normal and Q-distribution normal. Under some further assumptions on Q, this real number can be shown to be computable. Note that by a result of Beros and Beros that there are basic sequences Q for which there are no computable Q-distribution normal numbers.

Furthermore, we provide an effective version of the classical hotspot theorem.

11:30-11:45

Number of additive perfect unary forms over quadratic fields

Magdaléna Tinková

(Czech Technical University, Prague)

Perfect forms over integers are related to the famous lattice sphere packing problem. However, it is not clear how to define them over general number fields, and in this talk, we will focus on their additive version. By the result of Yasaki, over quadratic fields, some specific elements originating from continued fractions are minimal vectors of perfect unary forms. Using properties of continued fractions, we provide a more precise estimate on the number of such forms and show that for every $n \in \mathbb{N}$, there exist infinitely many quadratic fields with exactly n homothety classes of perfect unary forms.

Ergodic Averages in Positive Characteristic Time

11:50-12:05

J. Hančl[1], R. Nair[1,2], J-L Verger-Gaugry[3] and M. Weber[4]
([1]Ostrava, [2]Liverpool, [3]Chambéry, [4]Strasbourg

Let $(X, \beta, \mu, (T_g)_{g \in \mathbb{F}_q[t]})$ be a measure preserving $\mathbb{F}_q[t]$-action on the set X. Suppose $f \in L^1(X, \beta, \mu)$ and let $A_N f(x) = \frac{1}{|G_N|} \sum_{n \in G_N} f(T_n x)$ $(N = 1, 2, \ldots)$. Here $G_N = \{n \in \mathbb{F}_q[t] : |n| < N\}$. The pointwise averages of the form $(A_{G_N} f(x))_{N>0}$ converge directly analogously to Birkhoff pointwise ergodic theorem. In fact both theorems are different special cases of the ergodic theorem for amenable group actions due to A. A. Templemann. There are aspects of the behavious of these ergodic averages that do not follow from what we know about Templemann averages. One, for instance is the study of ergodic square functions. Here we have to get into the arithmetic fine structure of the action. Something that can be proved using this approach is the following : Given a strictly increasing sequence of natural numbers $(N_k)_{k \geq 1}$, set

$$Sf(x) = (\sum_{k=1}^{\infty} |A_{N_{k+1}} f(x) - A_{N_k} f(x)|^2)^{\frac{1}{2}}.$$

Then there exists $C > 0$ such that given $\lambda > 0$ we have

$$\mu(x : Sf(x) > \lambda) \leq C \frac{||f||_1}{\lambda}.$$

This implies that if $r > 1$ we have $||Sf||_r \leq C \, ||f||_r$. This materially refines what can be done using Templemann theorem alone.

Ostrowski, q-rationals and Sturmian sequences

12:10-12:25

Yakob Kahane and Sébastien Labbé
(Université du Québec à Montréal)

We study Ostrowski's numeration systems for real and integer numbers arising from the q-analog of rational numbers introduced by Morier-Genoud and Ovsienko. We present these numeration systems both in terms of standard multiplicative coding sequences and new admissible sequences, together with a translation between the two perspectives. Finally, we provide an interpretation of the q-analog of rational numbers in terms of certain factors of Sturmian sequences.

14:15-14:40

Beta-expansions for families of Salem numbers

Kevin G. Hare and Liam Orovec

(University of Waterloo)

Let $\beta > 1$. Consider the expansion $x = \sum_{j=1}^{\infty} a_j \beta^{-j}$ with $a_j \in \{0, 1, \ldots, \lfloor \beta \rfloor\}$. We say the expansion is a *greedy expansion with base β* if the a_j are lexicographically maximal. Similarly, we say the expansion is a *lazy expansion with base β* if the a_j are lexicographically minimal.

We say β is a *Pisot number* if it is a real algebraic integer greater than 1 such that all its Galois conjugates are strictly less than 1 in absolute value. We say β is a *Salem number* if it is a real algebraic integer greater than 1 such that all its Galois conjugates are less than or equal to 1 in absolute value, and at least one conjugate is on the unit circle. It is well known that all Pisot numbers are limit points of Salem numbers.

Consider a families of Salem numbers $\beta_n \to \beta^*$ where β^* is a Pisot number. In this talk we give criteria for when the greedy (resp. lazy) expansion of 1 with base β_n is eventually periodic based upon properties of the greedy (resp. lazy) expansion of β^*.

14:45-15:00

Multiple tilings for random β-transformations

Sven van Golden

(Universiteit Leiden)

The study of multiple tilings that arise from substitutions and β-transformations has been ongoing since the seminal works of Rauzy in 1982 and Thurston in 1989. In 2012 it was shown by Kalle and Steiner that any (not necessarily greedy) β-transformation gives rise to a multiple tiling in the case where β is a Pisot unit. This was done by considering a geometrical construction for the domain of the natural extension of such a β-transformation. Certain slices of this domain then act as prototiles for an aperiodic multiple tiling of a given hyperplane. In this talk we present an extension of this idea to random β-transformations, which at each iteration apply one out of a fixed finite collection of β-transformations, each corresponding to the same Pisot unit β, uniformly at random. We provide a geometrical construction for the domain of the natural extension of such a random β-transformation in its skew product representation. We see that, under the condition that these β-transformations share a common Markov partition, this domain similarly gives rise to an aperiodic multiple tiling, with interesting geometrical properties. We will discuss, for instance, how certain fibres of the multiple tiling are again multiple tilings of a lower dimensional space, which analoguously correspond to different realisations of the random dynamics. In particular, the multiple tilings induced by the deterministic β-transformations are contained in that of the random β-transformation as fibres. As a consequence, this provides a direct link between the existence of a common Markov partition shared between β-transformations and the covering degrees of their respective multiple tilings. Additionally, analogous to the deterministic case, we discuss how the natural extension domain of the random β-transformation provides a classification of the purely periodic points under certain realisations of the random system.

Hausdorff and packing measure of some restricted digit sets

15:05-15:20

Daniel Ingebretson

(University of Illinois Chicago)

While Hausdorff and packing dimension of fractal sets are well studied, less is known about the Hausdorff and packing measure. We survey this question for fractal sets arising from digit restrictions in some numeration systems, namely radix and Luroth expansions.

When the conformal dimension of a self-affine sponge of Lalley type is zero

15:50-16:05

Shuqin Zhang

(Zhengzhou University)

It is well known that if a metric space is uniformly disconnected, then its conformal dimension is zero. In the talk we first give a characterization when a self-affine sponge of Lalley-Gatzouras type is uniformly disconnected. Then we show that a self-affine sponge of Lalley-Gatzouras type has conformal dimension zero if and only if it is uniformly disconnected.

Friday, June 5, 2026

Thue–Morse along the sequence of cubes

9:30-10:30

Lukas Spiegelhofer
(Technical University of Leoben, Austria)

Results for the behaviour of automatic sequences along polynomials of degree higher than two are sparse. We evaluate the Thue–Morse sequence 01101001... along the sequence of cubes, and prove that each of its two values es attained with asymptotic frequency equal to 1/2. Concerning polynomials of degree at least 3, lower bounds existed before, as well as results for the sum-of-digits function in "sufficiently large bases" q (depending on the polynomial). This result extends Mauduit and Rivat's breakthrough results on the sum of digits of squares and primes, and answers another case of the last Gelfond problem on the sum-of-digits function.

The regularity of languages generated by a greedy numeration system

11:00-11:25

Savinien Kreczman
(Université de Liège)

In this talk, we study *positional numeration systems* and whether they generate a *regular* language. In a positional numeration system, a sequence $(U_n)_{n\in\mathbb{N}}$ is given, then every number is decomposed as a sum of terms in U and a word that represents this number is deduced from this decomposition. The *language* L_U of such a system, i.e., the set of all representations, can be regular or not, depending on the sequence U.

In 1998, Hollander provided some necessary and some sufficient conditions on U for the language to be regular. He did so by leveraging a connection between numeration systems with a *dominant root* (i.e., such that the limit $\lim_{n\to\infty}\frac{U_{n+1}}{U_n}$ exists) and Rényi numeration systems. However, not every positional numeration system that has a regular language has a dominant root.

In our talk, we relax this hypothesis and provide a criterion for the regularity of L_U even when U has no dominant root. Our main tool is the introduction of *alternate bases*, a generalization of Rényi numeration systems that replaces them when the dominant root condition is dropped. This allows us to reuse some of Hollander's lemmas in our more general setting, and to finely study the algorithm that represents numbers.

Joint work with Émilie Charlier.

11:30-11:45

Open dynamical systems in topologically expansive decreasing Lorenz maps

Yun Sun

Yiming Ding[1],Wolfgang Steiner[2], Yun Sun[3]

(Wuhan University of Science and Technology[1], Paris, IRIF[2], Wuhan University of Technology[3]) Let f be a topologically expansive decreasing Lorenz map and c be the critical point. The survivor set is denoted as $S_f(H) := \{x \in [0,1] : f^n(x) \notin H, \forall n \geq 0\}$, where H is an open subinterval. Here we focus on the hole $H = (a,b)$ with $a \leq c \leq b$ and $a \neq b$. By combinatorial tools, we obtain the admissible condition for the kneading invariants of expansive decreasing Lorenz maps. Moreover, let a be fixed, when f has an ergodic a.c.i.m., we prove that the topological entropy function $\lambda_f(a) : b \mapsto h_{top}(f|S_f(a,b))$ is a devil's staircase. At the special case that f being a negative β-transformation, using the Ledrappier-Young formula, the Hausdorff dimension function $\eta_f(a) : b \mapsto \dim_{\mathcal{H}}(S_f(a,b))$ is a devil's staircase. All the results can be extended to the case that b is fixed.

11:50-12:05

Numeration systems with a Cantor basis for the integers: the cost function

Georges Grekos

(Université de Saint-Étienne)

The numeration systems considered in this work have as *generating set* the so called *Cantor* generating set $\{b_0 = 1, b_1 = k_1, \dots, b_{i+1} = b_i k_{i+1}, \dots\}$ where $(k_i)_{i \geq 1}$ is a given sequence of integers with $k_i \geq 2$ for all $i \geq 1$. A *digit* c_i, $i \geq 0$, represents the number of repetitions of the element b_i of the generating set.

The digits will be integers subjected to certain conditions. In our context, c_i may be negative. The system will be *complete* and *non redundant*: every integer $n \in \mathbf{Z}$ will have a unique representation $n = \sum_{i=0}^{\infty} c_i b_i$ where all, except finitely many, c_i's are zero and each $c_i \in \mathbf{Z}$ depends on k_{i+1}.

The quantity $C(n) = \sum_{i=0}^{\infty} |c_i|$ corresponds to the "sum of the digits function" for systems with nonnegative digits; it is called *cost function*.

There are many works on the cost function when $k_1 = \cdots = k_n = \cdots = k$.

We present in this work a way to obtain, together with the uniqueness of representation, minimal cost for every n when the parity, even or odd, of each number k_i is arbitrary.

Joint work with Isao Wakabayashi (Tokyo)

Erdős–Wintner theorems for greedy numeration systems: linear recurrent bases and Cantor numeration

12:10-12:25

Johann Verwee

The classical Erdős-Wintner theorem gives a necessary and sufficient criterion for the existence of a limiting distribution of an additive arithmetical function, in terms of the convergence of two canonical series built from its local values at prime powers. In this talk, I discuss analogues of this phenomenon for additive functions attached to greedy numeration systems.

I will first present an effective Erdős-Wintner theorem for Cantor numeration systems. In this setting, every integer admits a unique expansion with respect to a sequence of place-values $(q_j)_{j \geqslant 0}$, and a Cantor-additive function decomposes as a sum of local digit contributions. I will explain a two-series criterion for the existence of a limit law in full generality, together with the associated infinite-product formula for the limiting characteristic function. I will also describe a trailing-window method that yields explicit bounds in Kolmogorov distance.

I will then turn to greedy numeration systems generated by increasing linear recurrences. In contrast with the Cantor case, the digits are no longer independent, but they satisfy finite-memory admissibility constraints encoded by a finite automaton. This leads to a transfer-matrix approach and to an Erdős-Wintner theorem for linearly recurrent bases, where the limit law is again characterized by the convergence of two canonical series. This framework contains Zeckendorf numeration as a special case.

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